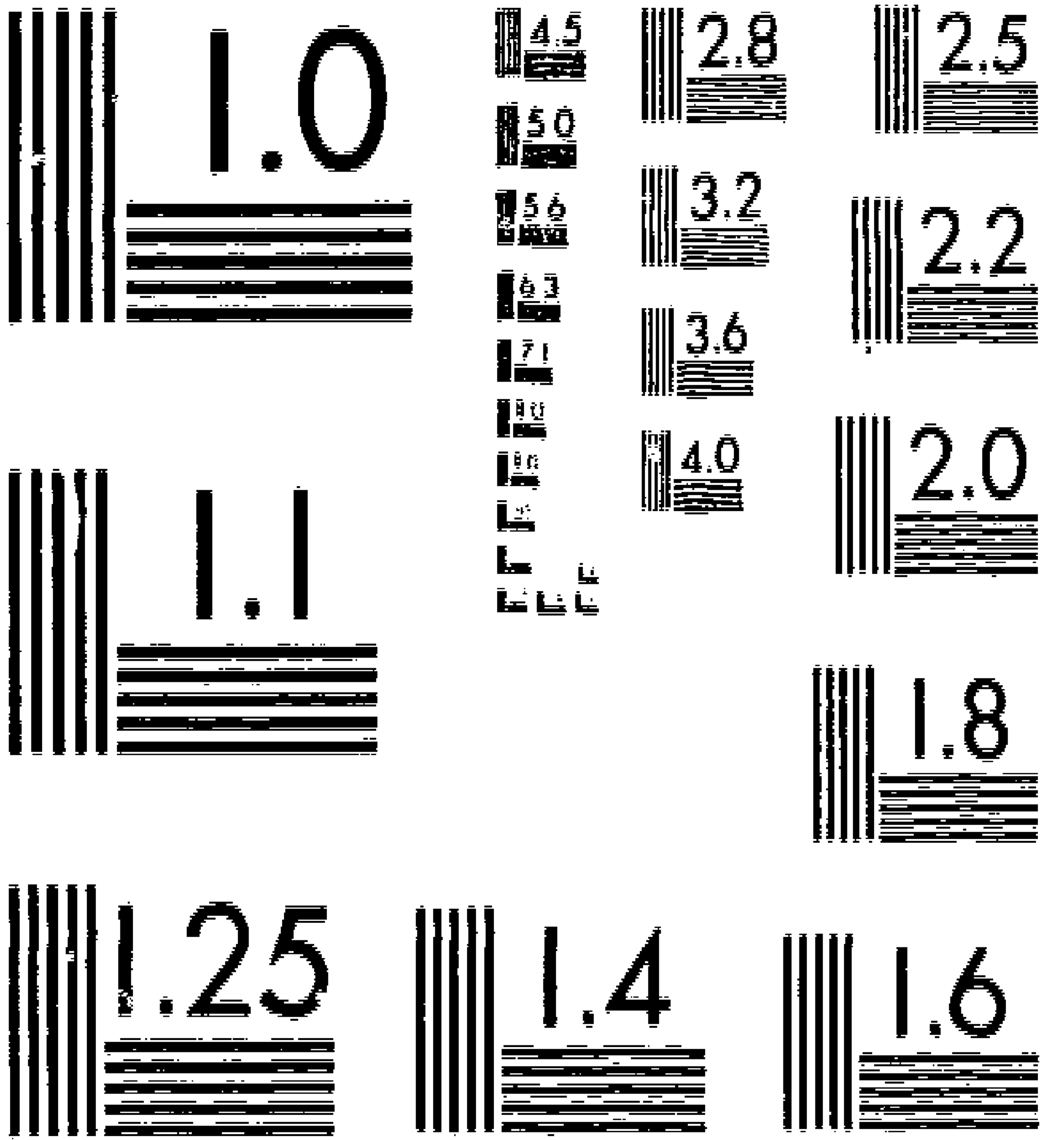

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ABSTRACT

This study tested the hypotheses that, following training in concept learning strategies, subjects would tend to follow the strategy taught and would perform better than untrained subjects. A sample of 60 graduate students was randomly assigned to three groups. The experimental task, administered via a computer teletype terminal, required subjects to find the arithmetic rule by which a given number was derived from three other given numbers. Subjects in the first group were taught a focussing strategy; the second group were taught a scanning strategy; and the third (control) group were taught no strategy, but used the same two practice problems as the other two groups. All subjects then worked on five experimental problems. The degree of focussing, purity of strategy, and number of trials to criterion were obtained from the computer record and analyzed by an analysis of variance and a multiple comparison test. Results showed that subjects taught conservative focussing showed the most focussing, but that subjects taught successive scanning also showed more focussing than the control group; the conservative focussing group used the purest strategy, but the successive scanning group was no purer than the control group; and neither experimental group was significantly more efficient at finding the rule than the control group. It is suggested that unmeasured personality factors may have been responsible for the unexpected results obtained. (MM)

STRATEGIES FOR LEARNING MATHEMATICAL CONCEPTS

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Paper presented at

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Subjects were taught focussing or scanning strategies for acquiring pseudo-mathematical concepts. Objective determination of degree of focussing and purity of strategy used indicated that instruction in use of a strategy did affect the way in which concepts were acquired. Results were discussed regarding implications for education and indications of further research needed.

Introduction

Mathematicians are employing a growing array of specialized computer-based systems to help them solve problems and gain new insights into mathematical relationships (Smith, 1970). So far, little has been done to make similar systems available to mathematics students in schools. Most of the efforts to introduce computers into schools have sought to cast the machines in the role of teachers or to promote problem solving through programming. Another approach is to provide systems which will aid in the solution of problems but which will not require the problem solver to learn how to program a computer. Such systems may be of great use in allowing students to discover mathematical concepts in a relatively short time through an investigative process. However, if such tools are provided to students, should the students be left to evolve their own way of using the systems or should they be taught strategies for investigation?

Strategies which subjects use in concept learning tasks have been studied often since Bruner, Goodnow, and Austin (1956) published their major work in this area. In most studies the experimenters have made no attempt to influence the strategies used by subjects. Scandura and Durnin (1968) and Scandura, Woodward, and Lee (1968) taught what was essentially one strategy at three different levels of generality. The strategy concerned a way to play a variation of the game of NIM. Other studies which involved teaching subjects a single strategy were conducted by Klausmeier and Meinke (1968) and Kornreich (1968, 1969). Tagatz (1967), and Tagatz, Walsh, and Layman (1969) taught two different strategies but the strategies were closely related.

Other than the latter two studies, there is no evidence in the literature of attempts to instruct subjects in the use of concept learning strategies with an objective assessment of whether the taught strategies were actually used. The present study initiates such an investigation.

Definition of "Concept"

Many researchers in the area of concept learning leave the word "concept" undefined. However, for most laboratory studies the term is defined at least implicitly. Glaser (1968) and Hunt (1962) seem to agree with Hunt, Marin, and Stone (1966) who stated, "A concept is a decision rule which, when applied to the description of an object, specifies whether of not a name may be applied (p. 10)." The rationale of studies by Klausmeier and Meinke (1968), Kornreich (1969), Suppes (1966), and Bruner, Goodnow, and Austin (1956) agrees with this characterization of a concept as a categorization rule.

Strategies for Concept Learning

Bruner, Goodnow, and Austin (1956) identified four strategies used by subjects in learning conjunctive concepts in a selection situation. Conjunctive concepts are specified by the presence of two or more attributes. A selection situation is one in which the subject selects exemplars from the universe as opposed to a reception situation in which the experimenter presents the next instance to the subject.

The strategies are: 1) simultaneous scanning, 2) successive scanning, 3) conservative focussing, and 4) focus gambling. The strategies are fully described by Bruner, Goodnow, and Austin (1956). It is sufficient in the present context to note that scanning strategies involve attempting to determine complete hypotheses as to the nature of a concept while focussing strategies involve paying attention to attributes to determine which are relevant. The studies reported by Tagatz (1967) and Tagatz, Walsh, and Layman (1969) taught two different strategies both of which concentrated on attributes. In the present study some subjects were taught a focussing strategy while others learned a scanning strategy.

Subjects

Sixty graduate students in education served as subjects on a voluntary basis. Subjects were assigned to three treatment groups of twenty each by a random procedure. One subject did not complete any of the experimental tasks.

Experimental Task

The task used for the experiment was adapted from an example of Scandura (1968). The task is intended to occupy a middle ground between the abstractness and artificiality of classical concept learning tasks and the familiarity of classroom mathematical concepts which are very difficult to control for prior learning or influences of unequally mastered prerequisite concepts. Thus the task is still an artificial, laboratory problem but it is hoped that it is more closely related to classroom concepts than are most traditional concept learning tasks.

The new task involves an ordered triple of digits and a number produced in some manner from them.

For example, (4,1,3):4

The concept to be acquired is the rule by which the number after the colon is produced from the digits inside the brackets. The subject searches for the rule by constructing his own triple and result. In each case he is informed whether or not the result he proposed may be produced from the triple he proposed by the rule he is trying to find.

Rules involve two digits which are combined by addition, subtraction, or multiplication. Digits are chosen from the triple according to their position in the triple, their relative magnitudes, or both. Thus, some typical rules with results to illustrate are:

First digit added to third digit (5,2,8):13

Smallest digit subtracted from second digit (1,3,7):2

Largest digit multiplied by middle-sized digit (9,4,6):54

Treatments

The independent variable in the study was instruction given to subjects in the use of strategies to acquire concepts. Two strategies were taught.

One strategy was essentially a conservative focussing strategy. As mentioned above, conservative focussing amounts to concentrating on discovering the relevant attributes of a concept. The complete concept is delineated only after the relevant attributes are found. For this reason, this strategy will be referred to as the attribute strategy (AS).

The second strategy was equivalent to successive scanning. This involves finding the concept by testing complete hypotheses. Thus, this strategy will be referred to as the hypothesis strategy (HS).

The third treatment condition was a control group (C) which received no strategy-related instruction.

The AS subjects were instructed first to determine which digits of the given triple were used to form the result. For example, if the triple and result

(4,1,3):3

is presented, the result may be produced either by multiplying the 1 and the 3, or by subtracting the 1 from the 4. AS subjects were instructed to change either the 4 or the 3 and nothing else. If, for example,

(5,1,3):3

were tried, a positive response indicates that the 1 and the 3 were used to form the result while a negative response indicates that the 4 and the 1 were used. From this point on all that needs to be determined is whether the digits used were employed because of position in the triple or magnitude.

To accomplish this, a return may be made to the originally given triple and result. Each of the digits to be investigated may be interchanged in turn with the one digit which is not used in forming the result. Thus, in exactly three trials, any rule may be determined.

The HS subjects were instructed to begin by determining which digits were actually used as were the AS subjects. Once this is determined there are only four possible rules. Each is tested directly as a complete unit. Some skill is needed in constructing trial instances to make sure that each tests only the one hypothesis which the subject has in mind at that time. This strategy may take up to four trials to determine a rule but it can conceivably lead to concept acquisition in two trials. Thus the HS is, on the average, as effficient as the AS.

Hypotheses

Hypothesis 1: AS subjects will use trials indicating more focussing than C subjects who, in turn, will focus more than HS subjects.

Hypothesis 2: AS and HS subjects will not differ significantly in the purity of strategies used but both will use strategies significantly purer than the strategies of C subjects.

Hypothesis 3: AS and HS subjects will not differ significantly in the number of trials used to acquire concepts but both groups will use significantly fewer trials than C subjects.

The first two hypotheses are claims that the teaching of strategies will be effective as shown by an objective measure of the use of strategies. Two different indicators will show type of strategy used and the degree of use, respectively.

The third hypothesis is based on the intuitive notion that any effective strategy that is learned will improve performance. It also reflects that there is no intuitive or theoretical basis for expecting one strategy to prove superior to the other.

Procedure

The entire experiment was designed to be computer administered. However, the computer aspects of the study, while very interesting, are not essential. Reference will be made to computer use only when such emphasis seems a significant aspect of the procedure. A full account of the computer aspects of the experiment is given by Durell (1970).

Subjects, regardless of specific treatment, all received the same basic instruction in the nature of the task. Each subject entered a room containing a standard teletype terminal. The computer was located in another room. He was told that all the instructions for the experiment would be given to him by means of the teletype and that he should respond using the teletype keyboard. Then the experimenter started execution of the experimental program and left the room. Instructions were typed out by the teletype. Periodic checks were made by the experimenter to ensure that the machinery was functioning properly.

The subject was told that the task would be to determine the rule by which a number was produced from a triple of digits. Each result was produced from two digits of the triple by additiion, subtraction, or multiplication. Numbers might be selected from the triple according to position or magnitude. The following example was given:

(4,1,3):3

A number of sample rules for this example were suggested.

At this point treatments differed. The strategy groups received instructions with examples in the use of one of the strategies to solve each of the following problems:

(5,2,6):7

(2,3,8):6

The control group received no instruction in use of a strategy but had the opportunity to practice the solution of the same two problems. The number of trials allowed on these practice problems was limited to twenty. If the subject did not know the rule after twenty trials, he was told the correct rule. Trials on the experimental problems were not limited.

The rules for the practice problems were:

First digit added to the second digit

First digit subtracted from the largest digit.

All subjects received the same instructions concerning the test items. That is, when a subject thought that he knew the rule for a problem, he could ask for a test. This procedure avoided the necessity for subjects to verbalize concepts and for a computer program to analyze typed-in statements of concepts. On the test he was presented with prepared triples for which he was required to provide the results. Giving the correct result for five consecutive test items is taken as evidence that the rule was known. Test items were constructed to minimize the chance of the wrong rule being accepted.

The experimental problems and rules were:

(4,1,5):4 First digit multiplied by second digit

(3,8,2):5 Middle-sized digit subtracted from largest digit

(9,3,6):3 Second digit subtracted from third digit

(4,2,3):6 Largest digit added to second digit

(1,5,2):3 First digit added to middle-sized digit.

All subjects were allowed a maximum of one hour to work on the experimental problems. There was no time limit for inidividual problems other than the over all one hour limit. Predictably, not all subjects finished the same number of problems.

Determination of Strategies Used

Most studies which purport to examine the strategies which subjects actually use in concept learning have used largely subjective determinations of the strategies. Byers (1967) has offered a means by which an objective measure of the degree of focussing which subjects employ can be determined. The method consists of comparing the instances proposed by the subject with the initially given focus instance. The number of dimensions on which the proposed instance differs from the focus instance is counted. A perfect use of a conservative focussing strategy would yield a difference of one on each instance. A simultaneous scanning strategy in which the subject consistently varied two attributes of his instances from the focus would give difference scores of two for each instance. Thus the mean of these differences would give a useful measure of the degree to which the subject was focussing with a lower score being indicative of more focussing. Also it is clear that if a strategy is used with high consistency the variance of the difference scores will be low so the variance may be used as a measure of the purity of the strategy employed, again with a low value indicating more purity.

Data Analysis

The hypotheses were tested by using multiple comparison techniques to detect differences between the means. As a first step, a one-way analysis of variance was carried out on the data for each problem to determine if significant differences existed among the means. It would have been preferrable to have taken problems as factors and done a two-way anlysis of variance but this approach was precluded by the fact that not all subjects finished the same number of problems within the time allowed. In any case, the analysis conducted would not bias the results toward producing spurious significance but would render the significant differences harder to detect.

The analysis of variance on degree of focussing, purity of strategy, and trials to criterion indicated that further analysis of the degree of focussing and purity of strategy data would be in order. Since simple differences of means for all treatments were desired, the Tukey method for multiple comparisons was used. In order to use the Tukey method, it was necessary to equalize the number of subjects in the compared groups which involved the random selection of some observations to be deleted as suggested by Glass and Stanley (1970).

Results

Analysis of variance of the degree of focussing, as shown in Table 1, indicated that there were significant differences among the mean focussing scores of the treatment groups on all of the first four problems at the .001 level. Table 2 shows that significant differences among the mean scores for purity of strategy used by the treatment groups were found for the first two and last two problems with significance levels ranging from .10 to .01. Table 3 indicates that no significant differences were found for mean trials to criterion for any of the five experimental problems.

Insert Tables 1, 2, and 3 about here

From the degree of focussing and purity of strategy use data, the Tukey method was used to produce a quotient associated with each difference of means. These quotients were compared with percentile points in the Studentized range distribution with J and J(n-1) degrees of freedom where J is the number of groups and n is the sample size for each group. A quotient larger than the percentile point of

Table 1
One-way Analysis of Variance
of Degree of Focussing
for Equalized Groups

Problem	Source of Variation	df	MS	F
1	Between groups	2	18.03	25.73 *
	Within groups	51	.71	
2	Between groups	2	15.53	19.37 *
	Within groups	51	.80	
3	Between groups	2	20.12	19.91 *
	Within groups	51	1.01	
4	Between groups	2	15.03	9.83 *
	Within groups	42	1.53	
5	Between groups	2	4.27	2.04
	Within groups	30	2.09	

* $p < .001$

Table 2
One-way Analysis of Variance
of Purity of Strategy
for Equalized Groups

Problem	Source of Variation	df	MS	F
1	Between groups	2	.97	2.95 *
	Within groups	51	.33	
2	Between groups	2	1.12	2.71 *
	Within groups	51	.41	
3	Between groups	2	.49	.88
	Within groups	51	.55	
4	Between groups	2	2.06	5.49 ***
	Within groups	42	.38	
5	Between groups	2	2.17	4.32 **
	Within groups	30	.50	

* $P < .10$
** $P < .025$
*** $P < .01$

Table 3
One-way Analysis of Variance
of Trials to Criterion

Problem	Source of Variation	df	MS	F
1	Between groups	2	17.61	1.35
	Within groups	54	13.07	
2	Between groups	2	82.39	3.17
	Within groups	54	26.00	
3	Between groups	2	23.91	2.72
	Within groups	54	8.78	
4	Between groups	2	95.47	.80
	Within groups	48	118.69	
5	Between groups	2	57.62	.98
	Within groups	34	58.63	

the Studentized range distribution indicated a significant difference of the associated means. The results of these analyses are shown in Tables 4 and 5. Of course, the calculations were only made for the problems on which the analysis of variance indicated a significant difference of the means.

Insert Tables 4 and 5 about here

By the prediction implicit in Hypothesis 1, the focussing scores of AS subjects were expected to be lower than those of the C subjects which would be lower than those of the HS subjects. The data gathered indicated that, in fact, AS subjects did focus more than the other subjects. However, HS subjects showed more focussing in the instances they generated than did the C subjects. Thus Hypothesis 1 was only partially supported.

Hypothesis 2 predicted that both strategy-trained treatments would produce purer strategy use than no training. It was found that AS subjects did use purer strategies than C subjects but no evidence indicated that HS subjects used purer strategies than C subjects. Thus Hypothesis 2 was also supported only in part.

There was no support forthcoming from the data for the contention of Hypothesis 3 that strategy-trained subjects would acquire the concepts more efficiently than untrained

Table 4
Summary of Tukey Method
of Multiple Comparisons of Means
for Degree of Focussing

Problem	n	$\sqrt{MS_W/n}$	$\bar{X}_A$	$\bar{X}_C$	$\bar{X}_H$	$\frac{\bar{X}_A-\bar{X}_C}{\sqrt{MS_W/n}}$	$\frac{\bar{X}_A-\bar{X}_H}{\sqrt{MS_W/n}}$	$\frac{\bar{X}_C-\bar{X}_H}{\sqrt{MS_W/n}}$
1	18	.197	1.32	3.30	2.04	-10.02***	-3.65*	6.38***
2	18	.211	1.56	3.42	2.60	-8.78***	-4.93***	3.85**
3	18	.237	1.74	3.85	2.68	-8.91**	-3.99**	4.92***
4	15	.319	1.08	3.07	2.24	-6.24**	-3.65*	2.59

* P<.05
** P<.025
*** P<.005

Table 5
Summary of Tukey Method
of Multiple Comparisons of Means
for Purity of Strategy

Problem	n	$\sqrt{MS_W/n}$	$\overline{X}_A$	$\overline{X}_C$	$\overline{X}_H$	$\frac{\overline{X}_A-\overline{X}_C}{\sqrt{MS_W/n}}$	$\frac{\overline{X}_A-\overline{X}_H}{\sqrt{MS_W/n}}$	$\frac{\overline{X}_C-\overline{X}_H}{\sqrt{MS_W/n}}$
1	18	.135	.265	.696	.626	-3.20*	-2.68	.52
2	18	.151	.354	.800	.769	-2.95*	-2.75	.20
4	15	.158	.288	1.03	.727	-4.66***	-2.77	1.89
5	11	.214	.064	.815	.851	-3.51**	-3.68**	.17

* P<.10
** P<.05
*** P<.01

subjects.

Discussion

The results indicate that it is possible to influence the strategies which subjects employ in a concept learning situation by instruction. However, it was expected that the C subjects would show a full range from focussing to non-focussing behaviour and thus that their average amount of focussing would fall between that of the AS and HS subjects. That this did not occur could be explained by the fact that C subjects were given no indication that there might exist efficient strategies for acquiring the concepts while both AS and HS subjects were informed that such strategies do exist. It may be that the mere knowledge of the existence of such strategies would make the strategy-trained subjects take a more purposeful approach to the problem solving than the C subjects.

The fact that HS subjects did not use purer strategies than C subjects was also unexpected. This might be caused by the trained groups acquiring only a superficial grasp of the strategies taught or by the fact that C subjects had a chance to practice the solution of two problems in the introductory part of the experiment while the AS and HS subjects had the more passive experience of seeing the same problems solved for them by a method which exemplified the appropriate strategy. That is, a practice effect might have been operating to give some advantage to C subjects which would obscure the effects of strategy training. Such a practice effect could also explain the reversal in the ordering of C and HS subjects in degree of focussing from that expected.

Either superficial learning of taught strategies or a practice effect working for the C subjects could account for the fact that trained subjects proved no more efficient than the untrained C group.

Personality Factors in Concept Learning

While it seems worthwhile to investigate teaching subjects different strategies to learn concepts, it is not expected that any one strategy will be of optimum value for all subjects. Various factors will contribute to differential abilities to use strategies. Some of these factors are likely to be dimensions of personality. One such dimension is impulsivity-reflectivity as defined by Kagan (1965). Kagan's classifications were made for school children with tests inappropriate for mature subjects. Measures of response latencies were collected in this experiment but did not provide a useful basis for classifying subjects. This was predictable as Kagan (1965) reported an increasing tendency for reflectivity with increasing age. Graduate students could well represent a set of reflective people.

Further Studies

This study has shown that the strategy using behaviour of mature subjects can be influenced by instruction. There seems to be a good possibility that the effects of the experiment were obscured by an unfortunate choice of experimental population and/or the difference in the training given to the experimental groups. Obvious corrections to these problems would be to rerun the experiment with a different population, perhaps junior high school students, and to redesign the strategy training to make it more interactive and thus more nearly equalize the training of the AS, HS, and C groups.

Further investigation of the degree to which strategy instructions can influence the behaviour of subjects should be of value in planning ways to make use of the calculating power of computer-based instructional systems. Such systems can be more than poorly executed teacher surrogates. Computer aids can speed up discovery learning experiences to make them practical in areas where they have been too time-consuming in the past to let them be of any practical value.

If such devices are to be developed and used effectively, research will be needed into strategies to use to obtain maximum gain from the new aids. This experimenter maintians a strong suspicion that interaction of strategies and personality factors will have to be considered.

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